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Guiding an adaptive system through chaos

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Abstract: *We study the parametric controls of self-adjusting systems with numerical models. We investigate the situation where the target dynamics changes slowly and passes through a chaotic region. We find that feedback destabilizes controls if the target is chaotic. If the control is unstable the system migrates to the closest non-chaotic target, i.e. it adapts to the edge of chaos. For weak controls the deviation between system dynamics and target is larger, but the system dynamics is less chaotic and therefore more predictable.*

I. INTRODUCTION

> *"Chaos is inevitable. In the sense that perturbation is evolutionary, it's also desirable. But managing it is essential. It's no use for any of us to hope that someone else will do it. Do you have your own personal strategies in place?"*
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> *C. P. Brinkworth, 2006 [1]*

Chaos means that plans go wildly astray. For instance in a situation with opposing opinions, strong emotions, and high stakes small events can have large unexpected consequences [2]. This behavior is called deterministic chaos [3,4]. Without management, deterministic chaos can produce arbitrary outcomes; some may be very negative such as missed deadlines or cost overruns, and some may be very positive, such as emergent leadership or identifying a strength. For instance, the evolution of an organization is deterministic chaos, if it encourages thinking out of the box and implements these new ideas rapidly, such as Google's "Chaos by design" strategy [5]. If the management does a good job in prioritizing ideas for implementation the overall outcome is positive.

Long term predictions of deterministic chaos are hard, since even very small amounts of noise can change the motion significantly. Short term predictions and even medium term predictions of chaos are not as difficult, since the motion is governed by a deterministic equation, plus some small noise [6]. In contrast, controlling the chaotic motion of an agent is often easy, both short term and long term. Just apply a control force which is equal to the difference between the next state of the agent and the target, and it will go to the target. This requires predicting the next state, which is a short term prediction, and therefore possible for chaotic agents. This control algorithm would not work for a random motion, since random motion can not be predicted, not even for one time step. This chaos control algorithm was introduced by Hübler in 1989 [7] and since has been further developed and widely used [8,9].

Such control algorithms work well if the parameters of the dynamical system are stationary. However in many real systems, the parameters are not stationary but self-adjusting through feed back loops. Most social organizations are modular structures with numerous feedback loops. A supervisor in a company controls the daily activities of a group of people, but his overall behavior is influenced by feedback from the supervised people. Activated genes control the cell dynamics, but the activation of genes occurs through feedback from the cell dynamics. The citizens of a state have to obey the laws but can change the laws through legislature. In many self-adjusting systems the feedback occurs on a much larger time scale than the dynamics.

One of the most striking features of self-adjusting parameters is their tendency to avoid chaos. This phenomenon is called “adaptation to the edge of chaos”. The concept “adaptation to the edge of chaos” refers to the idea that many complex adaptive systems, including those found in biology, seem to naturally evolve toward a narrow regime near the boundary between order and chaos [10]. Packard [11, 12] illustrated that adaptation to the edge of chaos occurs in a population of cellular automata rules which optimize their performance with a genetic algorithm. Self-organized critically [13] in avalanche and earthquake models is believed to be a similar phenomenon. Models of coupled neurons with self-adjusting coupling strength have been found to exhibit robust synchronization and suppression of chaos as well[14]. The edge of chaos occupies a prominent position because it has been found to be not only the optimal setting for control of a system [15], but also an optimal setting under which a physical system can support primitive functions for computation [16,17]. Possibly the simplest models for adaptation to the edge of chaos are self-adjusting map dynamics [18]. The numerical findings have been confirmed experimentally with Chua’s circuit [19]. A theoretical analysis by Baym et al. [20] predicts the location of the narrow parameter regime near the boundary to which the system evolves.

An issue which has received little attention is the control of simple self-adjusting system. Networks of simple self-adjusting systems can be used to model complicated biological networks and social organizations. Therefore the understanding of control of simple self-adjusting systems will help to understand the control of systems of systems. Controlling self-adjusting systems is particularly challenging, since the control competes with the self-adjustment. The self-adjustment may resist controls with certain targets. For instance if the target is chaotic the control may be unstable, since the self-adjusting system has the tendency to avoid chaos.

In the following we discuss the management of some very simple deterministic chaotic agents which adjust their dynamics through feedback. The agents can be thought of as business units or other nonlinear dynamical systems. The chaotic agents are controlled by a linear parametric control, which could be a manager or a computer algorithm.

II. PARAMETRIC CONTROLS WITH TIME DEPENDENT TARGET APPLIED TO SELF-ADJUSTING AGENTS

The dynamics of an agent or some other dynamical system is modeled with a logistic map with wavelet filtered feedback,

$$x_{n+1} = F(x_n, a_n), \tag{1}$$

n=0,1,2,...,N-1, where x_n is the state at time step n. The dynamics of the agent parameter a_n is governed by a control of strength *c*, and feedback of strength *f*,

$$a_{n+1} = a_n + c\,(A_n - a_n) + f\,w(x_n, x_{n-1}, ...), \tag{2}$$

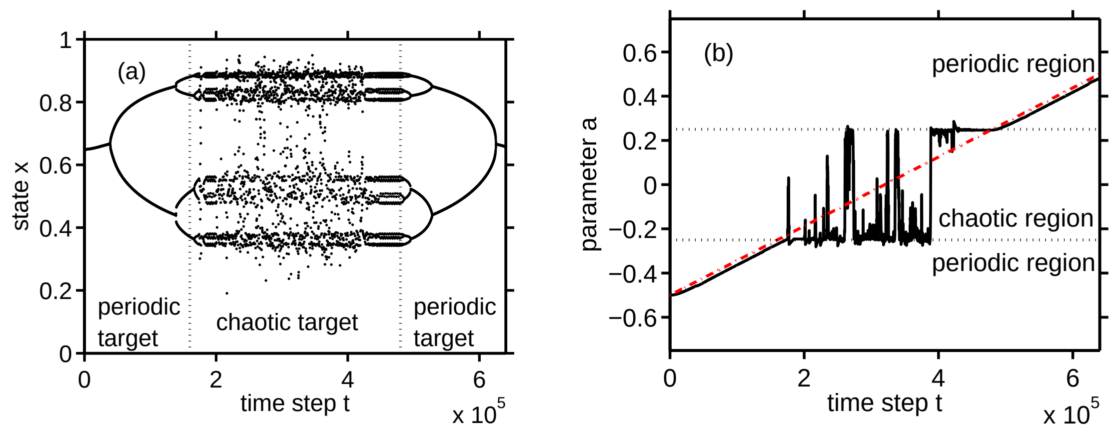
where $w(x_n, x_{n-1}, ...)$ is a wavelet filter[20]. The target of the control A_n is time dependent [21]:

$$A_{n+1} = A_n + r \tag{3}$$

where r is the rate of change of the target of the control. In the following we use a logistic map $F(x_n, a_n) = 3.8(1 - a_n^{\,2})\; x_n\,(1 - x_n)$, where $0 < x_n < 1$, $-1 < a_n < 1$, a Haar wavelet filter[21], $w = x_n - x_{n-32}$, and $A_0 = -0.5$ and $r = 0.0001$. For $-0.25 < a_n < 0.25$ the dynamics of the logistic map is mostly chaotic.

III. ADAPTATION TO THE EDGE OF CHAOS FOR SOFT CONTROLS

Figure 1 shows a numerical simulation for feedback strength $f = 0.2$ and a soft control, i.e. the strength of the control is small compared to the feedback, $c = 0.005 << f$. Figure 1a shows the state of the self-adjusting system versus time and Figure 1a shows its parameter value versus time. The dashed line in Figure 1b is the target parameter value of the linear control. The target starts out in the periodic regime, then passes through a chaotic regime, and finally reaches another periodic regime. Figure 1b shows that the system parameter is close to the target if the target is in the periodic regime. However if the target is in the chaotic regime, the control is unstable. In this case the parameter stays near the boundary of the chaotic regime, the edge of chaos. As long as the target is closer to the edge at $a = -0.25$, the parameter tends to be near this value. Later the parameter switches to the other edge at $a = 0.25$ as soon as the target gets closer to this value.



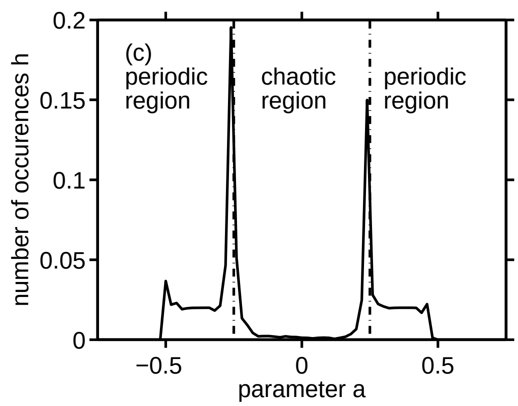
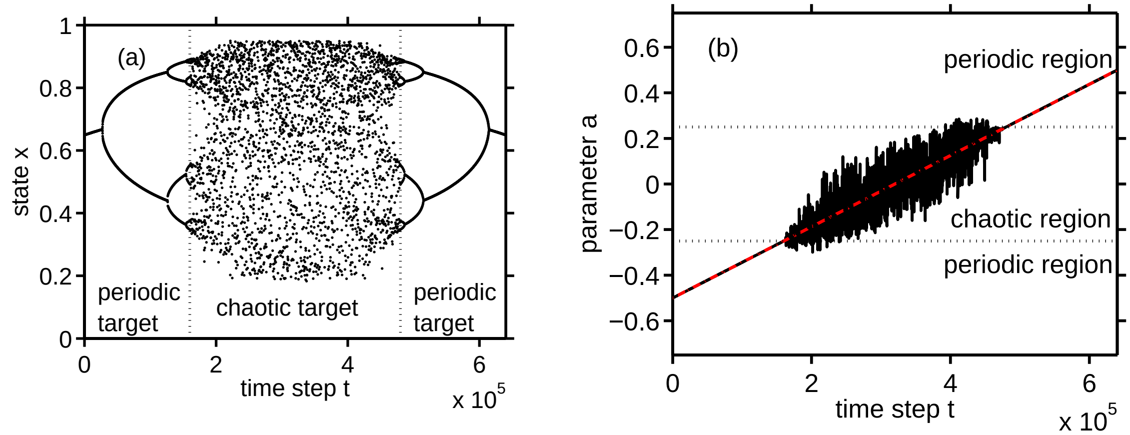


Figure 1. Dynamics of the self-adjusting system which is guided through a chaotic region by a soft control: the dynamics of the state (a), the dynamics of the parameter (b), and a histogram of the parameter values (c). The peaks in the histogram at the boundary of the chaotic region indicate that the system avoids chaos.

Therefore the histogram (Figure 1c) of the parameter values in Figure 1b has pronounced peaks at the two edges of chaos values, whereas the histogram values in the chaotic region are very small. This shows that the parameter avoids the chaotic region. This phenomenon is very similar to “adaptation to the edge of chaos” in systems without control.

IV. Stable control for strong controls

Figure 2 shows a numerical simulation for feedback strength $f = 0.2$ and a strong control, i.e. the strength of the control is larger than the feedback, $c = 0.5 > f$. Figure 2a shows the state of the self-adjusting system versus time and Figure 2b shows it’s parameter value versus time. The dashed line in Figure 2b is the target parameter value of the linear control. Figure 2b shows that the mean value of the system parameter is close to the target at all times, even if the target is in chaotic regime. However if the target is in the chaotic regime the fluctuations of the parameter are large. The onset of large fluctuations at the edge of chaos produces a peak in the histogram (Figure 2c) of the parameter values at the edge of chaos, but the peak is much less pronounced than in Figure 1. This shows that the parameter tends to avoid the chaotic region.



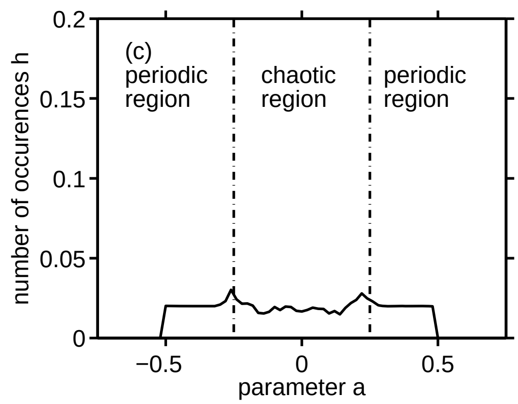


Figure 2. Dynamics of the self-adjusting system which is guided through a chaotic region by a strong control: the dynamics of the state (a), the dynamics of the parameter (b), and a histogram of the parameter values (c). The peaks in the histogram at the boundary of the chaotic region indicate are much less pronounced than in Figure 1.

The depleting of the histogram in the chaotic region occurs if the strength of the control is less than a critical c-value c_c. If the histogram value in the center of the chaotic regime drops more than *50%* the control is called unstable, and the corresponding *c*-value is defined to be the critical *c*-value. This critical strength of the control depends on the strength of the feedback. Figure 3 shows the critical strength c_c versus the feedback strength *f*. It illustrates the boundary between the area where the control is stable, and the area where adaptation to the edge of chaos is the dominant phenomena.

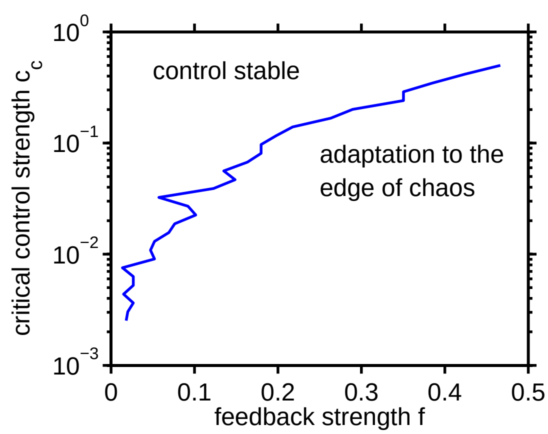


Figure 3. The boundary between the adaptation to the edge of chaos and stable control. In the area labeled "control stable" the parameter stays close to the target, even in the chaotic regime. In the area labeled "adaptation to the edge of chaos" the parameter stays outside the chaotic regime, even if the target is inside the chaotic regime.

V. Soft controls lead to better predictability

If the control is soft and adaptation to the edge of chaos occurs, this has advantages and disadvantages. First of all the control is small, i.e. generally requires fewer resources. Second, the dynamics avoids the chaotic regime, and is therefore more predictable. We measure the predictability with the Lyapunov exponent [6]

$$\lambda = \frac{1}{n_2 - n_1} \sum_{n=n_1}^{n_2} \ln | f'(x_n) |, \tag{4}$$

where n_1, n_2 are the time steps, when the target enters and exits the chaotic regime. The Lyapunov exponent is a measure for the predictability of the system. If the Lyapunov exponent describes how measurement uncertainties impact predictions. If the Lyapunov exponent is negative, the effect of measurement error decreases with the range of a prediction, whereas for systems with positive Lyapunov exponents the prediction error grows with the range of the prediction. Figure 4a shows that the Lyapunov exponent is negative for very small control and increases when the control is larger ($f = 0.2$). Soft controls have one mayor disadvantage; the deviation of the parameter from the target is large. We measure the deviation with the quantity:

$$\Delta a = \left(\frac{1}{n_2 - n_1} \sum_{n=n_1}^{n_2} (A_n - a_n)^2 \,|\right)^{1/2} \tag{5}$$

where n_1, n_2 are the time steps, when the target enters and exits the chaotic regime. Figure 2b shows the relation between the deviation and the control strength. The deviation decreases if the control strength is increased.

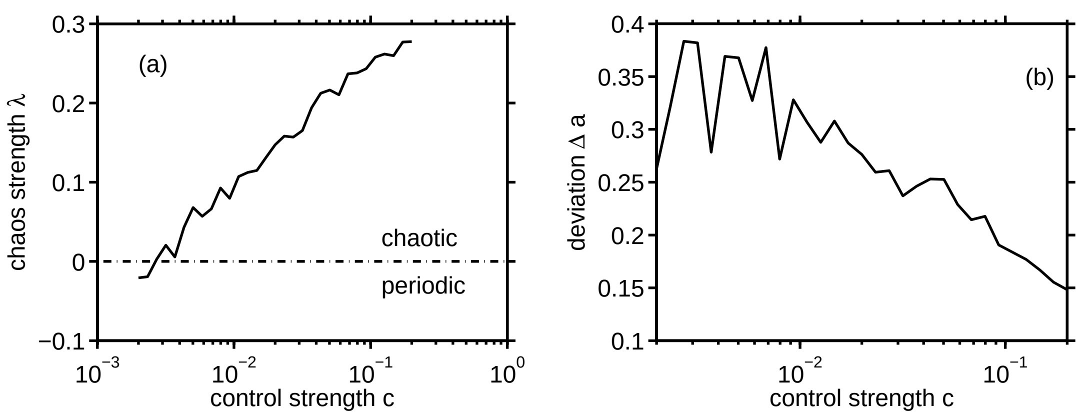


Figure 4. The strength of the chaos versus the strength of the control (a), and the deviation of the parameter from the target versus the control strength (b). If the control strength is small the deviation from the target is large but the system is more predictable..

VI. CONCLUSION

In summary, we study the parametric controls of self-adjusting systems with numerical models. There are three times involved: (i) the time scale of the dynamics of the state is unity; (ii) the time scale of the self-adjustment is one or two orders of magnitude longer, and (iii) the time scale at which the target of the control changes is much longer than that. Figure 1 shows that feedback destabilizes controls if the target is chaotic. Even if the control is unstable, both the dynamics of the state of the self-adjusting system and the evolution of it's parameter are both predictable. If the control is unstable the system parameter migrates to the closest non-chaotic target, i.e. it adapts to the edge of chaos. Figure 4 shows that for soft controls the deviation between system dynamics and target is larger, but the system dynamics is less chaotic and therefore more predictable.

This study assumes a separation of time scales by at least one order of magnitude. However we expect that similar features can be observed if the typical time scales are much closer, maybe only a factor of two apart. We expect that these features are also observable in networks of self-adjusting systems [22]. The numerical results in this paper are for the logistic map. Other work on adaptation to the edge of chaos has shown that the phenomena is applicable to a large set of mapping functions [18, 19, 21]. Therefore we expect similar results for other self-adjusting map dynamics.

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