Naïve Bayes Classifiers

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These notes assume you have already met Bayesian Networks.

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http://www.cs.cmu.edu/~awm/tutorials
Comments and corrections gratefully received.

For a more in-depth introduction to Naïve Bayes Classifiers and the theory surrounding them, please see Andrew’s lecture on Probability for Data Miners.
A simple Bayes Net

<table>
<thead>
<tr>
<th>J</th>
<th>Person is a Junior</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>Brought Coat to Classroom</td>
</tr>
<tr>
<td>Z</td>
<td>Live in zipcode 15213</td>
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<td>R</td>
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What parameters are stored in the CPTs of this Bayes Net?
A simple Bayes Net

\[
P(J) = \]

| \(J\) | Person is a Junior |
| \(C\) | Brought Coat to Classroom |
| \(Z\) | Live in zipcode 15213 |
| \(R\) | Saw “Return of the King” more than once |

\[
P(C|J) = \quad P(Z|J) = \quad P(R|J) =
\]

\[
P(C|\sim J) = \quad P(Z|\sim J) = \quad P(R|\sim J) =
\]

Suppose we have a database from 20 people who attended a lecture. How could we use that to estimate the values in this CPT?
A simple Bayes Net

\[ P(J) = \]

Person is a Junior

\[ \frac{\text{# people who walked to school}}{\text{# people in database}} \]

\[ \frac{\text{# people who walked to school and brought a coat}}{\text{# people who walked to school}} \]

\[ P(C|J) = \]

\[ P(C|\sim J) = \]

Suppose we have a database from 20 people who attended a lecture. How could we use that to estimate the values in this CPT?

\[ P(R|J) = \]

\[ P(R|\sim J) = \]

\[ \frac{\text{# coat-bringers who didn't walk to school}}{\text{# people who didn't walk to school}} \]
A new person shows up at class wearing an “I live right above the Manor Theater where I saw all the Lord of The Rings Movies every night” overcoat.

What is the probability that they are a Junior?
Naïve Bayes Classifier Inference

\[
P(J \mid C^\perp Z^\perp R) = \frac{P(J^\perp C^\perp Z^\perp R)}{P(C^\perp Z^\perp R)}
\]

\[
= \frac{P(J^\perp C^\perp Z^\perp R)}{P(J^\perp C^\perp Z^\perp R) + P(\neg J^\perp C^\perp Z^\perp R)}
\]

\[
= \frac{P(C \mid J)P(\neg Z \mid J)P(R \mid J)P(J)}{P(C \mid J)P(\neg Z \mid J)P(R \mid J)P(J) + P(C \mid \neg J)P(\neg Z \mid \neg J)P(R \mid \neg J)P(\neg J)}
\]
The General Case

1. Estimate $P(Y=v)$ as fraction of records with $Y=v$

2. Estimate $P(X_i=u \mid Y=v)$ as fraction of “$Y=v$” records that also have $X=u$.

3. To predict the $Y$ value given observations of all the $X_i$ values, compute

$$Y^{\text{predict}} = \arg\max_v P(Y = v \mid X_1 = u_1 \cdots X_m = u_m)$$
Naïve Bayes Classifier

\[ Y_{\text{predict}} = \arg\max_{v} P(Y = v \mid X_1 = u_1 \cdots X_m = u_m) \]

\[ Y_{\text{predict}} = \arg\max_{v} \frac{P(Y = v \land X_1 = u_1 \cdots X_m = u_m)}{P(X_1 = u_1 \cdots X_m = u_m)} \]

\[ Y_{\text{predict}} = \arg\max_{v} \frac{P(X_1 = u_1 \cdots X_m = u_m \mid Y = v)P(Y = v)}{P(X_1 = u_1 \cdots X_m = u_m)} \]

\[ Y_{\text{predict}} = \arg\max_{v} P(X_1 = u_1 \cdots X_m = u_m \mid Y = v)P(Y = v) \]

\[ Y_{\text{predict}} = \arg\max_{v} P(Y = v) \prod_{j=1}^{n_y} P(X_j = u_j \mid Y = v) \]

Because of the structure of the Bayes Net
More Facts About Naïve Bayes Classifiers

• Naïve Bayes Classifiers can be built with real-valued inputs*

• Rather Technical Complaint: Bayes Classifiers don’t try to be maximally discriminative---they merely try to honestly model what’s going on*

• Zero probabilities are painful for Joint and Naïve. A hack (justifiable with the magic words “Dirichlet Prior”) can help*.

• Naïve Bayes is wonderfully cheap. And survives 10,000 attributes cheerfully!

*See future Andrew Lectures
What you should know

• How to build a Bayes Classifier
• How to predict with a BC